A Systematic Approach to Delimited Control with Multiple Prompts

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Introduction to control operators

Separating a redex from its evaluation context

$$1 + 2 + 3 \times 4$$

Separating a redex from its evaluation context

$$1 + 2 + 3 \times 4$$

$$1 + 2 + \square \qquad\qquad 3 \times 4$$

Labeling the evaluation context

$$1 + 2 + 3 \times 4$$

$$k : 1 + 2 + \square \qquad 3 \times 4$$

Labeling the evaluation context

$$1 + 2 + $$

$$k : 1 + 2 + \square \qquad ?$$

Labeling the evaluation context

$$1 + 2 + \text{call/cc}(\lambda k.3 \times (k\ 5))$$

$$1 + 2 + \square \qquad \text{call/cc}(\lambda k.3 \times (k\ 5))$$

$$\Downarrow$$

$$k : 1 + 2 + \square \qquad 3 \times (k\ 5)$$

$$1 + 2 + 3 \times (k\ 5)$$

$$1 + 2 + 3 \times \square \qquad k\ 5$$

$$\Downarrow$$

$$1 + 2 + 5$$

Labeling the evaluation context

$$1 + 2 + \text{call/cc}(\lambda k.3 \times (k\ 5))$$

$$1 + 2 + \Box \qquad \text{call/cc}(\lambda k.3 \times (k\ 5))$$

$$\Downarrow$$

$$k : 1 + 2 + \Box \qquad 3 \times (k\ 5)$$

$$1 + 2 + 3 \times (k\ 5)$$

$$1 + 2 + 3 \times \Box \qquad k\ 5$$

$$\Downarrow$$

$$1 + 2 + 5$$

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$$1 + 2 + 3 \times \square \qquad k\ 5$$

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Labeling the evaluation context

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$$1 + 2 + 3 \times \square \qquad k\ 5$$

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Labeling the evaluation context

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$$1 + 2 + \square \qquad\qquad \text{call/cc}(\lambda k.3 \times (k\ 5))$$

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$$k : 1 + 2 + \square \qquad\qquad 3 \times (k\ 5)$$

$$1 + 2 + 3 \times (k\ 5)$$

$$1 + 2 + 3 \times \square \qquad\qquad k\ 5$$

$$\Downarrow$$

$$1 + 2 + 5$$

Labeling the evaluation context

$$1 + 2 + \mathrm{call/cc}(\lambda k.3 \times (k\ 5))$$
$$\Downarrow$$
$$1 + 2 + 5$$

Formalized by Felleisen, Friedman, Kohlbecker:
A syntactic theory of sequential control (1987)

Parigot's $\lambda\mu$

$$1 + 2 + \text{call/cc}(\lambda k.3 \times (k\ 5))$$

$$[*]1 + 2 + \mu\alpha.[\alpha]3 \times (\mu\_.[\alpha]5)$$

$$[*]1 + 2 + \square \qquad \mu\alpha.[\alpha]3 \times (\mu\_.[\alpha]5)$$

$$\Downarrow$$

$$\alpha : [*]1 + 2 + \square \qquad [\alpha]3 \times (\mu\_.[\alpha]5)$$

$$[*]1 + 2 + 3 \times (\mu\_.[*]1 + 2 + 5)$$

$$[*]1 + 2 + 3 \times \square \qquad \mu\_.[*]1 + 2 + 5$$

$$\Downarrow$$

$$\_ : [*]1 + 2 + 3 \times \square \qquad [*]1 + 2 + 5$$

$$[*]1 + 2 + 5$$

Parigot's $\lambda\mu$

$$[*]1 + 2 + \mu\alpha.[\alpha]3 \times (\mu\_.[\alpha]5)$$

$$[*]1 + 2 + \Box \qquad \mu\alpha.[\alpha]3 \times (\mu\_.[\alpha]5)$$

$$\Downarrow$$

$$\alpha : [*]1 + 2 + \Box \qquad [\alpha]3 \times (\mu\_.[\alpha]5)$$

$$[*]1 + 2 + 3 \times (\mu\_.[*]1 + 2 + 5)$$

$$[*]1 + 2 + 3 \times \Box \qquad \mu\_.[*]1 + 2 + 5$$

$$\Downarrow$$

$$\_ : [*]1 + 2 + 3 \times \Box \qquad [*]1 + 2 + 5$$

$$[*]1 + 2 + 5$$

Parigot's $\lambda\mu$

$$[*]1 + 2 + \mu\alpha.[\alpha]3 \times (\mu\_.[\alpha]5)$$

$$[*]1 + 2 + \Box \qquad \mu\alpha.[\alpha]3 \times (\mu\_.[\alpha]5)$$

$$\Downarrow$$

$$\alpha : [*]1 + 2 + \Box \qquad [\alpha]3 \times (\mu\_.[\alpha]5)$$

$$[*]1 + 2 + 3 \times (\mu\_.[*]1 + 2 + 5)$$

$$[*]1 + 2 + 3 \times \Box \qquad \mu\_.[*]1 + 2 + 5$$

$$\Downarrow$$

$$\_ : [*]1 + 2 + 3 \times \Box \qquad [*]1 + 2 + 5$$

$$[*]1 + 2 + 5$$

Parigot's $\lambda\mu$

$$[*]1 + 2 + \mu\alpha.[\alpha]3 \times (\mu\_.[\alpha]5)$$

$$[*]1 + 2 + \Box \qquad \mu\alpha.[\alpha]3 \times (\mu\_.[\alpha]5)$$

$$\Downarrow$$

$$\alpha : [*]1 + 2 + \Box \qquad [\alpha]3 \times (\mu\_.[\alpha]5)$$

$$[*]1 + 2 + 3 \times (\mu\_.[*]1 + 2 + 5)$$

$$[*]1 + 2 + 3 \times \Box \qquad \mu\_.[*]1 + 2 + 5$$

$$\Downarrow$$

$$\_ : [*]1 + 2 + 3 \times \Box \qquad [*]1 + 2 + 5$$

$$[*]1 + 2 + 5$$

Parigot's $\lambda\mu$

$$[*]1 + 2 + \mu\alpha.[\alpha]3 \times (\mu\_.[\alpha]5)$$

$$[*]1 + 2 + \square \qquad \mu\alpha.[\alpha]3 \times (\mu\_.[\alpha]5)$$

$$\Downarrow$$

$$\alpha : [*]1 + 2 + \square \qquad [\alpha]3 \times (\mu\_.[\alpha]5)$$

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$$[*]1 + 2 + 3 \times \square \qquad \mu\_.[*]1 + 2 + 5$$

$$\Downarrow$$

$$\_ : [*]1 + 2 + 3 \times \square \qquad [*]1 + 2 + 5$$

$$[*]1 + 2 + 5$$

Parigot's $\lambda\mu$

Advantages:

- Well-behaved, fine-grained reduction theory
- A jump is not a function call
- Clearer "top level" of the program
- Foundations in classical logic

Delimited control

Delimited control

$$1 + \# 2 + \text{shift}(\lambda k.3 \times (k\ 5))$$

$1 + \# \square$ $2 + \square$ $\text{shift}(\lambda k.3 \times (k\ 5))$

Delimited control

$$1 + \#\,2 + \text{shift}(\lambda k.3 \times (k\ 5))$$

$$1 + \#\,\square \qquad 2 + \square \qquad \text{shift}(\lambda k.3 \times (k\ 5))$$

$\lambda\mu$ with 1 dynamic co-variable

$$1 + \#2 + \text{shift}(\lambda k.3 \times (k\ 5))$$

$$[*]1 + \mu\widehat{\text{tp}}.\,[\widehat{\text{tp}}]2 + \mu\alpha.[\widehat{\text{tp}}]3 \times (\mu\widehat{\text{tp}}.[\alpha]5)$$

$$[*]1 + \mu\widehat{\text{tp}}.\,\square \qquad [\widehat{\text{tp}}]2 + \square \qquad \mu\alpha.[\widehat{\text{tp}}]3 \times (\mu\widehat{\text{tp}}.[\alpha]5)$$

$$\Downarrow$$

$$[*]1 + \mu\widehat{\text{tp}}.\,\square \qquad \alpha :\ [\widehat{\text{tp}}]2 + \square \qquad [\widehat{\text{tp}}]3 \times (\mu\widehat{\text{tp}}.[\alpha]5)$$

$$[*]1 + \mu\widehat{\text{tp}}.[\widehat{\text{tp}}]3 \times (\mu\widehat{\text{tp}}.[\widehat{\text{tp}}]\,2 + 5\,)$$

$$\Downarrow$$

$$[*]1 + \mu\widehat{\text{tp}}.[\widehat{\text{tp}}]3 \times\ (\mu\widehat{\text{tp}}.\,[\widehat{\text{tp}}]\,7\ )$$

$$[*]1 + \mu\widehat{\text{tp}}.[\widehat{\text{tp}}]3 \times\ \square \qquad \mu\widehat{\text{tp}}.\,\square \qquad [\widehat{\text{tp}}]7$$

$$\Downarrow$$

$$[*]1 + \mu\widehat{\text{tp}}.[\widehat{\text{tp}}]3 \times 7$$

$\lambda\mu$ with 1 dynamic co-variable

$$[*]1 + \mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]2 + \mu\alpha.[\widehat{\mathsf{tp}}]3 \times (\mu\widehat{\mathsf{tp}}.[\alpha]5)$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.\Box \qquad [\widehat{\mathsf{tp}}]2 + \Box \qquad \mu\alpha.[\widehat{\mathsf{tp}}]3 \times (\mu\widehat{\mathsf{tp}}.[\alpha]5)$$

$$\Downarrow$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.\Box \qquad \alpha : [\widehat{\mathsf{tp}}]2 + \Box \qquad [\widehat{\mathsf{tp}}]3 \times (\mu\widehat{\mathsf{tp}}.[\alpha]5)$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]3 \times (\mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]\,2 + 5\,)$$

$$\Downarrow$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]3 \times (\mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]\,7\,)$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]3 \times \Box \qquad \mu\widehat{\mathsf{tp}}.\Box \qquad [\widehat{\mathsf{tp}}]7$$

$$\Downarrow$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]3 \times 7$$

$\lambda\mu$ with 1 dynamic co-variable

$$[*]1 + \mu\widehat{tp}.\,[\widehat{tp}]2 + \mu\alpha.[\widehat{tp}]3 \times (\mu\widehat{tp}.[\alpha]5)$$

$$[*]1 + \mu\widehat{tp}.\Box \qquad [\widehat{tp}]2 + \Box \qquad \mu\alpha.[\widehat{tp}]3 \times (\mu\widehat{tp}.[\alpha]5)$$

$$\Downarrow$$

$$[*]1 + \mu\widehat{tp}.\Box \qquad \alpha : [\widehat{tp}]2 + \Box \qquad [\widehat{tp}]3 \times (\mu\widehat{tp}.[\alpha]5)$$

$$[*]1 + \mu\widehat{tp}.[\widehat{tp}]3 \times (\mu\widehat{tp}.[\widehat{tp}]\,2 + 5\,)$$

$$\Downarrow$$

$$[*]1 + \mu\widehat{tp}.[\widehat{tp}]3 \times (\mu\widehat{tp}.[\widehat{tp}]\,7\,)$$

$$[*]1 + \mu\widehat{tp}.[\widehat{tp}]3 \times \Box \qquad \mu\widehat{tp}.\Box \qquad [\widehat{tp}]7$$

$$\Downarrow$$

$$[*]1 + \mu\widehat{tp}.[\widehat{tp}]3 \times 7$$

$\lambda\mu$ with 1 dynamic co-variable

$$[*]1 + \mu\widehat{\mathsf{tp}}.\,[\widehat{\mathsf{tp}}]2 + \mu\alpha.[\widehat{\mathsf{tp}}]3 \times (\mu\widehat{\mathsf{tp}}.[\alpha]5)$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.\,\square \qquad [\widehat{\mathsf{tp}}]2 + \square \qquad \mu\alpha.[\widehat{\mathsf{tp}}]3 \times (\mu\widehat{\mathsf{tp}}.[\alpha]5)$$

$$\Downarrow$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.\,\square \qquad \alpha : [\widehat{\mathsf{tp}}]2 + \square \qquad [\widehat{\mathsf{tp}}]3 \times (\mu\widehat{\mathsf{tp}}.[\alpha]5)$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]3 \times (\mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]\,2+5\,)$$

$$\Downarrow$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]3 \times (\mu\widehat{\mathsf{tp}}.\,[\widehat{\mathsf{tp}}]\,7\,)$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]3 \times \square \qquad \mu\widehat{\mathsf{tp}}.\,\square \qquad [\widehat{\mathsf{tp}}]7$$

$$\Downarrow$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]3 \times 7$$

$\lambda\mu$ with 1 dynamic co-variable

$$[*]1 + \mu\widehat{\mathsf{tp}}.\,[\widehat{\mathsf{tp}}]2 + \mu\alpha.[\widehat{\mathsf{tp}}]3 \times (\mu\widehat{\mathsf{tp}}.[\alpha]5)$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.\Box \qquad [\widehat{\mathsf{tp}}]2 + \Box \qquad \mu\alpha.[\widehat{\mathsf{tp}}]3 \times (\mu\widehat{\mathsf{tp}}.[\alpha]5)$$

$$\Downarrow$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.\Box \qquad \alpha : [\widehat{\mathsf{tp}}]2 + \Box \qquad [\widehat{\mathsf{tp}}]3 \times (\mu\widehat{\mathsf{tp}}.[\alpha]5)$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]3 \times (\mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]\,2 + 5\,)$$

$$\Downarrow$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]3 \times (\mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]\,7\,)$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]3 \times \Box \qquad \mu\widehat{\mathsf{tp}}.\Box \qquad [\widehat{\mathsf{tp}}]7$$

$$\Downarrow$$

$$[*]1 + \mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]3 \times 7$$

The dynamic nature of $\widehat{\text{tp}}$

Decomposing the CPS

- Have (Ariola, Herbelin, Sabry. HOSC 2009):

$$\lambda\mu\widehat{\mathrm{tp}} \xrightarrow{CPS^2} \lambda$$

- Goal:

$$\lambda\mu\widehat{\mathrm{tp}} \xrightarrow{CPS} \lambda\widehat{\mathrm{tp}} \xrightarrow{EPS} \lambda$$

- So that $CPS^2 = EPS \circ CPS$
- What is $\lambda\widehat{\mathrm{tp}}$?

First attempt: Ordinary dynamic binding

Example $[\widehat{\mathrm{tp}}]\,\mu\widehat{\mathrm{tp}}.\,[\widehat{\mathrm{tp}}]x \rightarrow [\widehat{\mathrm{tp}}]\,x$

$$[\![[\widehat{\mathrm{tp}}]\mu\widehat{\mathrm{tp}}.[\widehat{\mathrm{tp}}]x]\!] = \textbf{dlet}\ \widehat{\mathrm{tp}} = (\lambda y.\widehat{\mathrm{tp}}\ y)\ \textbf{in}\ \widehat{\mathrm{tp}}\ x$$

Semantics given by Moreau (HOSC 1998):

$$\begin{aligned}
&\textbf{dlet}\ \widehat{\mathrm{tp}} = (\lambda y.\widehat{\mathrm{tp}}\ y)\ \textbf{in}\ \widehat{\mathrm{tp}}\ x \\
\rightarrow\ &\textbf{dlet}\ \widehat{\mathrm{tp}} = (\lambda y.\widehat{\mathrm{tp}}\ y)\ \textbf{in}(\lambda y.\widehat{\mathrm{tp}}\ y)\ x \\
\rightarrow\ &\textbf{dlet}\ \widehat{\mathrm{tp}} = (\lambda y.\widehat{\mathrm{tp}}\ y)\ \textbf{in}\ \widehat{\mathrm{tp}}\ x \\
\rightarrow\ &\ldots
\end{aligned}$$

First attempt: Ordinary dynamic binding

Example

$$[\widehat{\mathsf{tp}}]\,\mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]x \to [\widehat{\mathsf{tp}}]x$$

$$[\![[\widehat{\mathsf{tp}}]\mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]x]\!] = \mathbf{dlet}\, \widehat{\mathsf{tp}} = (\lambda y.\widehat{\mathsf{tp}}\ y)\, \mathbf{in}\, \widehat{\mathsf{tp}}\ x$$

Semantics given by Moreau (HOSC 1998):

$$\begin{aligned} & \mathbf{dlet}\, \widehat{\mathsf{tp}} = (\lambda y.\widehat{\mathsf{tp}}\ y)\, \mathbf{in}\, \widehat{\mathsf{tp}}\ x \\ \to\ & \mathbf{dlet}\, \widehat{\mathsf{tp}} = (\lambda y.\widehat{\mathsf{tp}}\ y)\, \mathbf{in} (\lambda y.\widehat{\mathsf{tp}}\ y)\ x \\ \to\ & \mathbf{dlet}\, \widehat{\mathsf{tp}} = (\lambda y.\widehat{\mathsf{tp}}\ y)\, \mathbf{in}\, \widehat{\mathsf{tp}}\ x \\ \to\ & \ldots \end{aligned}$$

Second attempt: One-shot dynamic binding

Example

$$[\widehat{\mathrm{tp}}]\,\mu\widehat{\mathrm{tp}}.[\widehat{\mathrm{tp}}]x \rightarrow [\widehat{\mathrm{tp}}]x$$

$$[\![[\widehat{\mathrm{tp}}]\mu\widehat{\mathrm{tp}}.[\widehat{\mathrm{tp}}]x]\!] = \mathbf{dlet}\,\widehat{\mathrm{tp}} = (\lambda y.\widehat{\mathrm{tp}}\ y)\,\mathbf{in}\,\widehat{\mathrm{tp}}\ x$$

$$\begin{aligned} &\mathbf{dlet}\,\widehat{\mathrm{tp}} = (\lambda y.\widehat{\mathrm{tp}}\ y)\,\mathbf{in}\,\widehat{\mathrm{tp}}\ x \\ &\rightarrow (\lambda y.\widehat{\mathrm{tp}}\ y)\ x \\ &\rightarrow \widehat{\mathrm{tp}}\ x \end{aligned}$$

Decomposing the CPS

- $\lambda\widehat{\text{tp}}$: single one-shot dynamic variable
- Have (Ariola, Herbelin, Sabry. HOSC 2009):

$$\lambda\mu\widehat{\text{tp}} \xrightarrow{CPS^2} \lambda$$

- Also have:

$$\lambda\mu\widehat{\text{tp}} \xrightarrow{CPS} \lambda\widehat{\text{tp}} \xrightarrow{EPS} \lambda$$

- So that $CPS^2 = EPS \circ CPS$

Delimited control with multiple prompts

“Easy” vs. “hard” effects in delimited control

- Filinski (POPL ’94): shift+reset simulate all monadic effects
- Easy:
  - exceptions (of one type)
  - state (one reference)
  - non-determinism
- Harder:
  - exceptions (of multiple type)
  - state (many references)
  - lazy evaluation (with multiple bindings)

Extension: multiple named prompts

Delimited control with multiple prompts

$$\#^{\widehat{\alpha}}1 + \#^{\widehat{\beta}}2 + \#^{\widehat{\delta}}3 + \mathsf{shift}^{\widehat{\beta}}(\lambda k.t)$$

$$\#^{\widehat{\alpha}}1 + \#^{\widehat{\beta}}\Box \qquad 2 + \#^{\widehat{\delta}}3 + \Box \qquad \mathsf{shift}^{\widehat{\beta}}(\lambda k.t)$$

Delimited control with multiple prompts

$$\#^{\widehat{\alpha}} 1 + \#^{\widehat{\beta}} 2 + \#^{\widehat{\delta}} 3 + \mathsf{shift}^{\widehat{\beta}}(\lambda k.t)$$

$$\#^{\widehat{\alpha}} 1 + \#^{\widehat{\beta}} \square \qquad 2 + \#^{\widehat{\delta}} 3 + \square \qquad \mathsf{shift}^{\widehat{\beta}}(\lambda k.t)$$

Simple extension: Multiple dynamic variables

- Extend intermediate language with multiple dynamic variables
- CPS the same, only EPS is changed
- Let's us implement (multi-type) exceptions
- But, ...

Simple extension: Multiple dynamic variables

μ only captures its immediate context (up to $[\widehat{\delta}]$)

$$\mu\widehat{\alpha}.[\widehat{\alpha}]1 + \mu\widehat{\beta}.[\widehat{\beta}]2 + \mu\widehat{\delta}.[\widehat{\delta}]3 + \mu\alpha.[\widehat{\beta}]\boldsymbol{t}$$

$$\mu\widehat{\alpha}.[\widehat{\alpha}]1 + \mu\widehat{\beta}.[\widehat{\beta}]2 + \mu\widehat{\delta}.\Box \qquad [\widehat{\delta}]3 + \Box \qquad \mu\alpha.[\widehat{\beta}]\boldsymbol{t}$$

How can we capture dynamically-bound contexts?

Splitting the dynamic environment

Two options to split the dynamic environment:

- Change the behavior of μ
- Leave μ as it is and add another operator

Splitting the dynamic environment

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- Change the behavior of μ
- Leave μ as it is and add another operator

Splitting the dynamic environment

New command: $\mu^2\Delta\uparrow^{\widehat{\beta}}.t$

- Search for the dynamically nearest binding of $\widehat{\beta}$
- Give the prefix of dynamic bindings leading to $\widehat{\beta}$ the label Δ
- Evaluate t in the context formerly bound to $\widehat{\beta}$

Splitting the dynamic environment

$$\mu\widehat{\alpha}.[\widehat{\alpha}]1 + \mu\widehat{\beta}.[\widehat{\beta}]2 + \mu\widehat{\delta}.\mu^2\Delta \uparrow^{\widehat{\beta}}.t$$

$$\mu\widehat{\alpha}.[\widehat{\alpha}]1 + \Box \qquad \mu\widehat{\beta}.[\widehat{\beta}]2 + \mu\widehat{\delta}.\Box \qquad \mu^2\Delta \uparrow^{\widehat{\beta}}.t$$

$$\Downarrow$$

$$\mu\widehat{\alpha}.[\widehat{\alpha}]1 + \Box \qquad \Delta : [\widehat{\beta}]2 + \mu\widehat{\delta}.\Box \qquad t$$

$$\mu\widehat{\alpha}.[\widehat{\alpha}]1 + t$$

Splitting the dynamic environment

$$\mu\widehat{\alpha}.[\widehat{\alpha}]1 + \mu\widehat{\beta}.[\widehat{\beta}]2 + \mu\widehat{\delta}.\,\mu^2\Delta\uparrow^{\widehat{\beta}}.t$$

$$\mu\widehat{\alpha}.[\widehat{\alpha}]1 + \square \qquad \mu\widehat{\beta}.[\widehat{\beta}]2 + \mu\widehat{\delta}.\square \qquad \mu^2\Delta\uparrow^{\widehat{\beta}}.t$$

$$\Downarrow$$

$$\mu\widehat{\alpha}.[\widehat{\alpha}]1 + \square \qquad \Delta : [\widehat{\beta}]2 + \mu\widehat{\delta}.\square \qquad t$$

$$\mu\widehat{\alpha}.[\widehat{\alpha}]1 + t$$

Summary I

- Fine-grained, backwards-compatible reduction theory for delimited control with multiple prompts
  - Dybvig, Peyton Jones, Sabry: A monadic framework for delimited continuations (2007)
  - Gunter, Rémy, Riecke: A generalization of exceptions and control in ML-like languages (1995)
- Matching CPS transform and operational semantics

Summary II

- Formalized the dynamic nature of $\widehat{\text{tp}}$
  - Kiselyov, Shan, Sabry: Delimited dynamic binding (2006)
- Clarified the behavior of "naked" delimited control
  - What is the behavior of shift outside of a reset?
  - What is the meaning of $[\widehat{\text{tp}}]5$ when $\widehat{\text{tp}}$ is unbound?

Questions?

Unbound $\widehat{\text{tp}}$

Expressiveness

Unbound $\widehat{tp}$

Making the metacontext explicit

- Extend $\lambda\mu$ with 2^{nd}-rder commands and co^2-constant(s):
  - $\bullet$: Metacontext in which $\widehat{\text{tp}}$ is unbound
  - $\circledast$: Metacontext in which $\widehat{\text{tp}}$ is bound to $*$
- Errors depend on initial conditions

Unbound $\widehat{\mathsf{tp}}$

Is shift$(\lambda_.9)$ an error?

$$[\bullet][*]\mu\_.[\widehat{\mathsf{tp}}]9 \to [\bullet][\widehat{\mathsf{tp}}]9$$

$$[\circledast][*]\mu\_.[\widehat{\mathsf{tp}}]9 \to [\circledast][\widehat{\mathsf{tp}}]9$$

$$[\bullet][*]\mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]t = [\circledast][\widehat{\mathsf{tp}}]t$$

Unbound $\widehat{\mathsf{tp}}$

Is shift($\lambda k.k$ 9) an error?

$$[\bullet][*]\mu\alpha.[\widehat{\mathsf{tp}}]\mu\widehat{\mathsf{tp}}.[\alpha]9 \to [\bullet][\widehat{\mathsf{tp}}]\mu\widehat{\mathsf{tp}}.[*]9$$
$$\to [\bullet][*]9$$

$$[\bullet][*]\mu\alpha.\uparrow^{\widehat{\mathsf{tp}}} \mu\widehat{\mathsf{tp}}.[\alpha]9 \to [\bullet]\uparrow^{\widehat{\mathsf{tp}}} \mu\widehat{\mathsf{tp}}.[*]9$$
$$\not\to$$

Expressiveness: Encoding control operators via μ

Expressiveness I

$$\text{call/cc} = \lambda h.\mu\alpha.[\alpha]h\ (\lambda x.\mu\_.[\alpha]x)$$

Expressiveness II

$$\#t = \mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]t$$

$$\mathsf{shift} = \lambda h.\mu\alpha.[\widehat{\mathsf{tp}}]h\ (\lambda x.\mu\widehat{\mathsf{tp}}.[\alpha]x)$$

Expressiveness III

$$\#^{\widehat{\alpha}} t = \mu\widehat{\alpha}.[\widehat{\alpha}]t$$

$$\mathsf{abort}^{\widehat{\alpha}}\, t = \mu\_.[\widehat{\alpha}]t$$

Expressiveness IV

$$\#t = \mu\widehat{\mathsf{tp}}.[\widehat{\mathsf{tp}}]t$$

$$\mathsf{shift} = \lambda h.\mu\alpha.[\widehat{\mathsf{tp}}]h\ (\lambda x.\mu\widehat{\mathsf{tp}}.[\alpha]x)$$

$$\mathsf{shift}_0 = \lambda h.\mu\alpha. \uparrow^{\widehat{\mathsf{tp}}} h\ (\lambda x.\mu\widehat{\mathsf{tp}}.[\alpha]x)$$

Expressiveness V

$$\#^{\widehat{\alpha}} t = \mu\widehat{\alpha}.[\widehat{\alpha}]t$$
$$\mathsf{shift}^{\widehat{\alpha}} = \lambda h.\mu\beta.\mu^2\Delta \uparrow^{\widehat{\alpha}}.\mu\widehat{\alpha}.[\widehat{\alpha}]h\ (\lambda x.\mu\widehat{\alpha}.[\Delta][\beta]x)$$
$$\mathsf{shift_0}^{\widehat{\alpha}} = \lambda h.\mu\beta.\mu^2\Delta \uparrow^{\widehat{\alpha}}.h\ (\lambda x.\mu\widehat{\alpha}.[\Delta][\beta]x)$$