Efficient algorithms and Partial k-trees.

Motivation

Many types of restrictions on the class of all graphs have been investigated in order to deal with the inherent difficulty (NP-completeness) of some optimization and decision problems on graphs. Some of these restrictions have impact on the tractability of many different optimization and decision problems. The papers in this issue have the common theme that the restrictions studied relate a graph to a tree, i.e., they study classes of graphs that can be concisely represented by classes of finitely labeled trees. There are many ways of relating a graph to a tree, and many of them yield identical or similar classes of graphs. The title of this collection indicates one way of defining the connection between graphs and trees: partial k-trees are graphs embeddable in the k-trees, a family of graphs with a simple construction process based on trees. Another way to make the connection is the treewidth concept defined by Robertson and Seymour() - requiring a graph to be the non-disjoint union of small graphs that intersect in the pattern of a tree. If the small graphs are allowed to have at most $k+1$ vertices, the generated class is exactly the class of partial k-trees. The papers to follow also investigate graphs constructible by algebraic means, and by minimal forbidden minors - if one forbidden minor is planar, then the family defined has bounded tree-width.

The research area covered by these papers has a broad scope, from fundamental studies of asymptotic feasibility far removed from direct practical applicability, to real engineering applications in reliability engineering, operations research and artificial intelligence.

Paper overview

The articles in this volume deal with different aspects of graphs with bounded treewidth. The simplest class of those graphs, besides forests, are partial 2-trees and their subclasses. A class of graphs closely related to partial 2-trees is series-parallel graphs, well known by electrical engineers for at least a century. In fact, MacMahon in his 1892 paper could assume the utility of this class known by his readers and notes in passing that the Wheatstone bridge, known to graph theorists as K_4, is not a series-parallel graph. He did however not know that K_4 also completely characterizes a closely related family as a forbidden minor (or homeomorph). Such results came much later, inspired by Kuratowski's 1928 planar graph characterization. The paper by Korneënko makes the definitive connection between partial 2-trees and series-parallel graphs. That two-connected partial 2-trees are series-parallel has been well known but articulation vertices with more than two components caused formal discomfort. It turns out that already in 1984 Korneenko published a definition of generalized series-paralle graphs that fills the gap. He uses a Branch operation on two graphs in addition to the Series and Parallel operations. This paper is reprinted here with the author's permission.

An alternative view of bounded treewidth graph is that of k-terminal graphs. In this formalism, operations on graphs are defined through a set of recurrences. Thus, series-parallel graphs become 2-terminal graphs. Grinstead and Slater use that formalism to solve a set of domination problems with a single "template" of recurrence equations, where different problems require only a slight change in parameter values. Another subclass of series-parallel graphs is the class of outerplanar graphs. Syslo and Winter implicitly use the parse tree of such graphs to solve a problem related to domination: covering the graph by faces of its outerplane embedding.

The notion of obstructions (minimal forbidden minors) is indelibly connected with bounded treewidth by Robertson and Seymour's proof of Wagner's conjecture. Even though the obstruction set is finite for any minor-closed class of graphs, only few such sets have been constructed. In their paper, Kinnersley and Langston discuss the obstruction set for a subclass of partial 2-trees that exhibit a more "linear" structure. Those graphs are characterized by an important layout parameter of graphs namely their pathwidth. They describe the search for all the obstructions in terms of the equivalent Matrix Layout Problem and arrive at the set of 110 obstructions.

Another way of finite characterization of graphs with bounded treewidth is that of graph rewriting (reduction). Here, again, leaf pruning for trees and series and parallel reductions for partial 2-trees have been known as simple recognition procedures for these classes of graphs. To recognize partial 3-trees a slightly more discriminating "3-leaf pruning" adds three more reduction rules. Lagergren shows in his paper that such vertex reduction graph rewriting can not be used for general partial k-trees.

We know that many inherently difficult optimization problems have polynomial (or even linear) time solutions when their instances are restricted to partial k-trees. In many cases, the algorithms constructed by the general method, although polynomial, are of high polynomial degree or use very large static data tables (like state sets multiply exponential in k). Thus, algorithms solving difficult problems by less universal means are definitely of interest. In a short note, Hoover shows how the chromatic index problem can be solved efficiently on partial k-trees. A general paradigm for solving certain problems on partial k-trees is described by Mahajan and Peters in terms of table-driven algorithms based on the concept of local properties defining the problem. In their discussion, graphs with bounded treewidth are formally presented as k-terminal graphs, similarly to the treatment of Grinstead and Slater.

The paper by Bodlaender develops methods for constructing algorithms for graphs with bounded treewidth. One of these methods is an improvement of the self-reduction approach of Fellows. The other method is a novel use of Monadic Second Order formalization of graph properties in finding bounded vertex and edge sets.

The remaining two papers of this volume are concerned with problems somewhat beyond treewidth; in fact, generalizations thereof. Wanke describes an algebraic construction method where an unbounded number of new edges can be introduced by the composition operator. This turns out to yield a linear time decision algorithm for MS properties expressible without quantification over edge sets. Wanke goes further and develops polynomial time algorithms for properties not so expressible, Simple Max Cut and Hamiltonian Cycle. Courcelle's paper is one in a series of his papers investigating fundamental aspects of Monadic Second Order definitions of graph families. If a set L of graphs is defined as those graphs satisfying an MS formula (which may use quantification over edge sets) and L has uniform bound on vertex degree and on treewidth then L can be also defined without using quantification over edge sets. Other result states that the membership in L for graphs generated by various types of graph grammars can be decided in linear time. However, for graph grammars of the NL-type, the derivation tree must also be given.